IF $\vec{u} = \langle 3,-2,1\rangle$ AND $\vec{v} = \langle -2,-1,c\rangle$ AND $\vec{u} \perp \vec{v}$, FIND C

$$\vec{u} \perp \vec{v} \longrightarrow \vec{u}\cdot\vec{v} = 0$$

$$\langle 3,-2,1\rangle \cdot \langle -2,-1,c\rangle = 0$$

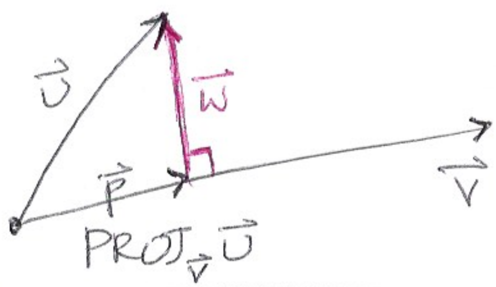
$$-6+2+c=0$$

$$c=4$$

PROJECTIONS ($\approx$ SHADOW)

VECTOR PROJECTION OF $\vec{u}$ ONTO $\vec{v}$

$$= \mathrm{PROJ}_{\vec{v}}\vec{u} = \frac{\vec{v}\cdot\vec{u}}{\vec{v}\cdot\vec{v}}\vec{v}$$



$$\vec{p} = k\vec{v}$$

$$\vec{p}\cdot\vec{w} = 0 \qquad (\vec{p} \perp \vec{w})$$

$$\vec{p}+\vec{w} = \vec{u}$$

$$k\vec{v}\cdot\vec{w} = 0 \longrightarrow k\vec{v}\cdot(\vec{u}-k\vec{v})=0$$

$$k\vec{v}+\vec{w}=\vec{u} \longrightarrow \vec{w} = \vec{u}-k\vec{v}$$

$$\vec{v}\cdot(\vec{u}-k\vec{v})=0$$

$$\vec{v}\cdot\vec{u} - \vec{v}\cdot k\vec{v} = 0$$

$$\vec{v}\cdot\vec{u} - k(\vec{v}\cdot\vec{v})=0$$

$$k = \frac{\vec{v}\cdot\vec{u}}{\vec{v}\cdot\vec{v}} \longrightarrow \vec{p} = \frac{\vec{v}\cdot\vec{u}}{\vec{v}\cdot\vec{v}}\vec{v}$$

PROPERTIES

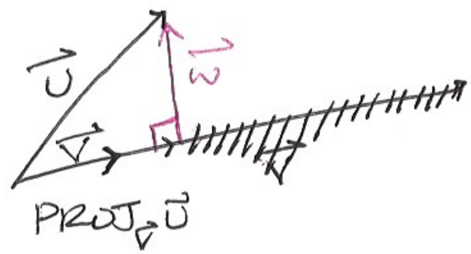
$$\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$$

$$k(\vec{u}\cdot\vec{v}) = k\vec{u}\cdot\vec{v} = \vec{u}\cdot k\vec{v}$$

$$(\vec{u}+\vec{v})\cdot\vec{w} = \vec{u}\cdot\vec{w}+\vec{v}\cdot\vec{w}$$

(VECTOR)
FIND THE PROJECTION OF $\vec{u} = \langle 2, 1, 3 \rangle$ ONTO $\vec{v} = \langle 1, 0, 2 \rangle$

$\text{PROJ}_{\vec{v}} \vec{u}$



$\vec{w}$ = ORTHOGONAL COMPONENT OF $\vec{u}$

$$\text{PROJ}_{\vec{v}} \vec{u} + \vec{w} = \vec{u}$$

$$\vec{w} = \vec{u} - \text{PROJ}_{\vec{v}} \vec{u}$$

$$\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v} = \frac{2+0+6}{1+0+4} \langle 1, 0, 2 \rangle$$

$$= \frac{8}{5} \langle 1, 0, 2 \rangle$$

$$= \left\langle \frac{8}{5}, 0, \frac{16}{5} \right\rangle$$

$$\vec{w} = \langle 2, 1, 3 \rangle - \left\langle \frac{8}{5}, 0, \frac{16}{5} \right\rangle$$

$$= \left\langle \frac{2}{5}, 1, -\frac{1}{5} \right\rangle$$

SANITY $\vec{w} \perp \vec{v}$

$$\vec{w} \cdot \vec{v} = \left\langle \frac{2}{5}, 1, -\frac{1}{5} \right\rangle \cdot \langle 1, 0, 2 \rangle$$

$$= \frac{2}{5} + 0 - \frac{2}{5} = 0 \checkmark$$

$$\vec{U}\cdot\vec{U} = \|\vec{U}\|\,\|\vec{U}\|\,\overset{1}{\cancel{\cos\theta}} = \|\vec{U}\|^2$$

$$\theta = 0$$

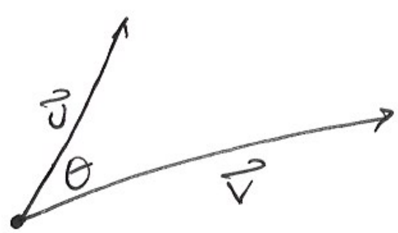
$$\text{PROJ}_{\vec{V}}\vec{U} = \frac{\vec{U}\cdot\vec{V}}{\vec{V}\cdot\vec{V}}\vec{V} = \frac{\vec{U}\cdot\vec{V}}{\|\vec{V}\|^2}\vec{V}$$

$$= \frac{\vec{U}\cdot\vec{V}}{\|\vec{V}\|}\,\frac{\vec{V}}{\|\vec{V}\|}$$

UNIT VECTOR
MAGNITUDE = 1

$$\|\text{PROJ}_{\vec{V}}\vec{U}\| = \frac{|\vec{U}\cdot\vec{V}|}{\|\vec{V}\|}$$ = SCALAR PROJECTION OF $\vec{U}$ ONTO $\vec{V}$

$$\vec{U}\cdot\vec{V} = \|\vec{U}\|\,\|\vec{V}\|\cos\theta \longrightarrow \theta = \cos^{-1}\frac{\vec{U}\cdot\vec{V}}{\|\vec{U}\|\|\vec{V}\|}$$



SAME INITIAL POINT

12.4 CROSS PRODUCT/VECTOR PRODUCT

$$\vec{U} \times \vec{V} = \langle U_2V_3 - U_3V_2,\ U_3V_1 - U_1V_3,\ U_1V_2 - U_2V_1 \rangle$$

IF $\vec{U} = \langle U_1, U_2, U_3 \rangle$

$\vec{V} = \langle V_1, V_2, V_3 \rangle$

OR

$$\vec{U} \times \vec{V} = \left| \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ U_1 & U_2 & U_3 \\ V_1 & V_2 & V_3 \end{array} \right| \begin{array}{cc} \vec{i} & \vec{j} \\ U_1 & U_2 \\ V_1 & V_2 \end{array}$$

$$= U_2V_3\vec{i} + U_3V_1\vec{j} + U_1V_2\vec{k}$$
$$- U_3V_2\vec{i} - U_1V_3\vec{j} - U_2V_1\vec{k}$$
$$= (U_2V_3 - U_3V_2)\vec{i} + (U_3V_1 - U_1V_3)\vec{j}$$
$$+ (U_1V_2 - U_2V_1)\vec{k}$$

FIND $\langle 2,-1,3\rangle \times \langle 1,-2,-1\rangle$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 3 \\ 1 & -2 & -1 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 2 & -1 \\ 1 & -2 \end{matrix} = \begin{matrix} \vec{i} + 3\vec{j} - 4\vec{k} \\ +6\vec{i} + 2\vec{j} + \vec{k} \end{matrix}$$

$$= \langle 7, 5, -3\rangle$$

MANDATORY SANITY CHECK

$$\boxed{\begin{matrix} \langle 7,5,-3\rangle \cdot \langle 2,-1,3\rangle = 14-5-9 = 0 \\ \langle 7,5,-3\rangle \cdot \langle 1,-2,-1\rangle = 7-10+3 = 0 \end{matrix}}$$

$$\boxed{\vec{u} \times \vec{v} \perp \vec{u}, \vec{v}}$$

FIND $\langle 1,-2,-1\rangle \times \langle 2,-1,3\rangle$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -1 \\ 2 & -1 & 3 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 1 & -2 \\ 2 & -1 \end{matrix} = \begin{matrix} -6\vec{i} - 2\vec{j} - \vec{k} \\ -\vec{i} - 3\vec{j} + 4\vec{k} \end{matrix} = \langle -7, -5, 3\rangle$$

$\vec{u} \times \vec{v} \neq \vec{v} \times \vec{u}$ NOT COMMUTATIVE

$$\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$$

$$k\vec{a}\times\vec{b} = \vec{a}\times k\vec{b} = k(\vec{a}\times\vec{b})$$

$$\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$$

$$(\vec{b}+\vec{c})\times\vec{a} = \vec{b}\times\vec{a}+\vec{c}\times\vec{a}$$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{c}$$

$$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$$

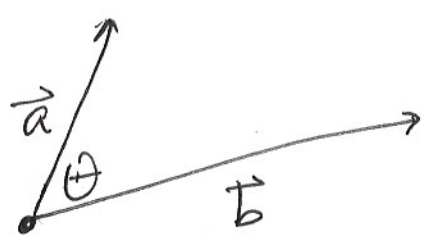
$$\vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})$$

TRY TO EXPAND $(\vec{a}\times\vec{b})\times\vec{c}$

$$= -\vec{c}\times(\vec{a}\times\vec{b})\ \ldots$$

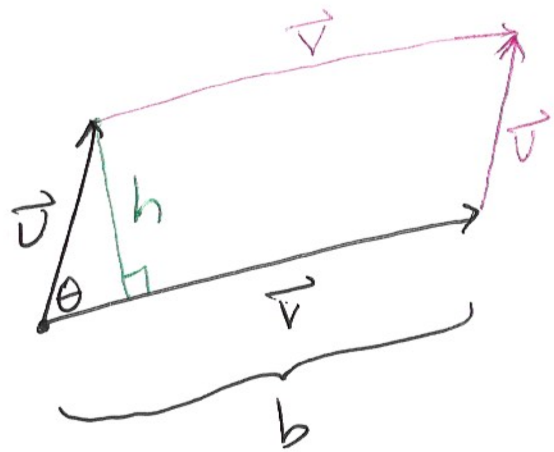
$$\|\vec{a}\times\vec{b}\| = \|\vec{a}\|\|\vec{b}\|\sin\theta$$

DON'T USE $\theta = \sin^{-1}\dfrac{\|\vec{a}\times\vec{b}\|}{\|\vec{a}\|\|\vec{b}\|}$!



USE RIGHT HAND RULE
TO DETERMINE DIRECTION OF $\vec{u} \times \vec{v}$

$$\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \|\vec{v}\| \sin\theta$$



AREA OF
PARALLELOGRAM WITH
$\vec{u}, \vec{v}$ AS ADJACENT EDGES

$$A = bh = \|\vec{v}\| \|\vec{u}\| \sin\theta$$
$$= \|\vec{u} \times \vec{v}\|$$

$$b = \|\vec{v}\|$$

$$\sin\theta = \frac{h}{\|\vec{u}\|}$$

$$h = \|\vec{u}\| \sin\theta$$